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Lecture Notes on Dynamic Adverse Selection

Bolton & Dewatripont, ch 9

The commitment problem

- Equilibrium in a static model of adverse selection: *separation*

- In a multi-period setting, separation by type early on problematic for the informed - unless the uninformed *can commit* to not taking advantage of the information that such separation reveals.

- Example: Insurance a la Rothschild and Stiglitz (1976)
    - Single period: high risks fully insured at high rate, low risks partially insured at low rate.
    - Multiple periods: high risks would prefer mimicking the low risk to buying the high risk contract and flagging their type for future transactions.

- Solution: *long-term contract*?
    - Subject to renegotiation – so the initial contract must be renegotiation proof
    - Renegotiation-proofness imposes constraints that are stricter than incentive constraints.

- Are long-term contracts feasible at all? Are they observed?

- *Short-term contracts* subject to the ratchet effect.
    - *Ratchet effect* etymology: discussion in the 1970s on the Soviet system, e.g. Weitzman (1976).

- Repeated contracting suffers from renegotiation and ratchet effects.
    - The uninformed party's per-period payoff is lower than what he would gain in a static model
    - Trading in an anonymous market would have been better.

Coasian dynamics

- The *Coase conjecture*: A monopolist selling a perfectly durable good would, with no discounting, be forced to sell at marginal cost.
  - Coase (1972)
  - A commitment problem – the monopolist is competing with his own future selves.

- Equivalent setting: bargaining with asymmetric information.

- Here: the analysis of Hart and Tirole (1988).

- Model: A buyer and a seller. A single unit of a good. One seller, one buyer. Two periods.
  - Buying in period 1 means consumption in periods 1 and 2.
  - Seller valuation 0. Buyer valuation per period v_H or v_L, $0 < v_L < v_H < 1$. $\Pr(v_H) = \beta$.
  - Common discount factor $\delta \leq 1$. Buyer valuation of two periods' consumption $v_i(1 + \delta) = v_i\Delta$.

- Linear model. Not much scope for screening. Two possible outcomes: either pooling, or separation with cutoff.

- Full commitment
  - Long-term contract – equivalent to the solution of the static model.
  - Two possible outcomes
    - Pooling: Both buyer types buy at price equal to low valuation $v_L(1 + \delta)$.
    - Separation with cutoff: High-type buyer buys at price $v_H(1 + \delta)$, while low type does not buy.
  - Focus on cutoff – closest resemblance with monopoly pricing. Assume H type sufficiently likely:

$$\beta > \beta^* := v_L / v_H.$$

The importance of commitment

- What if L type does not buy in period 1 but suggests to seller a transaction at a low price ($\leq v_L$) in period 2?
- If the seller is not committed to abstain from the temptation, the buyer, if she is high type, is not willing to accept the $v_H(1 + \delta)$ in period 1.

No commitment

- Selling without commitment
- Renting without commitment

Selling without commitment

- Only spot contracts feasible: price P_t in period t.
- If buyer buys in period 1, the game is over. If buyer does *not* buy in period 1, let $\beta(P_1)$ be the seller's probability assessment that buyer is H type.
- If $\beta(P_1) > \beta'$, then seller cuts off low type and sets $P_2 = v_H$; otherwise, the two types are pooled at $P_2 = v_L$.
- No surplus to L type in period 2 following no buy in period 1. Accordingly, in period 1, L type buyer accepts $P_1 \leq v_L\Delta$.
- What H type buyer does in period 1 depends on what she believes seller does in period 2 following no buy in period 1.
    - If she expects low price, $P_2 = v_L$, then she buys in period 1 if and only if $P_1 \leq P' = v_H\Delta - \delta(v_H - v_L) = v_H + \delta v_L$.

- Three possible strategies for seller
  1. Pooling: selling for sure in period 1 at $P_1 = v_L\Delta$.
  2. Full separation: selling in period 1 to H type, in period 2 to L type, with $P_1 = P'$ and $P_2 = v_L$. Revenue: $\beta P' + (1 - \beta)\delta v_L = \beta v_H + \delta v_L$. This dominates pooling when $\beta > \beta'$.
  3. Semiseparation: Selling in period 1 at an even higher price, $P_1 = v_H\Delta$. High type buyer indifferent between buying in periods 1 and 2. Seller indifferent in period 2 between v_H and v_L. This dominates full separation when β is close to 1, but the opposite holds when β is close to β'.

- Focusing on cases where $\beta > \beta'$, we see that
  - commitment leads to high price and a chance of no buy;
  - no commitment implies price decrease over time and purchase delayed.

- The ratchet effect – incentives for the good type not to reveal her information – not present here.

Renting without commitment

- Spot rental – an alternative to selling the good. Rental price R_t.

- Buyer anonymity: Seller face a continuum of anonymous buyers. Renting dominates selling. Selling implies moving down the demand curve over time. Renting implies meeting the same demand curve in each period, when buyers' purchase histories cannot be recorded.

- Selling without commitment: the seller's problem is not being able to commit not to lower price when buyer type is expected to be low.

- Renting without commitment: a second problem arises – not being able to commit not to increase price when buyer type is expected to be high; the ratchet effect.

- Period 2 as before.

- Period 1 – again three possible strategies for seller:
  1. Pooling: Rent to both types in period 1 and cutoff of low type in period 2 [since now $\beta(R_1) > \beta'$]. $R_1 = v_L$, $R_2 = v_H$. Revenue: $v_L + \delta\beta v_H$.
  2. Full separation: Cutoff in period 1. Incentive constraint for H type: $v_H - R_1 \geq \delta(v_H - v_L) \Rightarrow R_1 = (1-\delta)v_H + \delta v_L$. Revenue: $\beta[(1-\delta)v_H + \delta v_L] + \delta[\beta v_H + (1-\beta)v_L] = \beta v_H + \delta v_L$; dominates pooling when $\beta > \beta'$ and $\delta < 1$.
  3. Semiseparation: Essentially the same outcome as in the selling case.

- Renting and selling are similar in the two-period case.

- With more than two periods, there will limited amount of revelation of types before the last two periods.
  - In the three-period case, when β is greater than but close to β', selling with full separation is preferred to renting, when full separation is not feasible because of the ratchet effect.

- In summary
  - renting is preferred to selling when buyers are anonymous.
  - selling is preferred to renting with nonanonymous buyers, except in the two-period case when they are equivalent.

Renegotiation

- Suppose long-term contracts are feasible – but the seller is unable to commit not to offer the buyer a new contract at the beginning of period 2 that would replace the original contract if the buyer accepts.

- An intermediate case between full commitment and no commitment.
  - a realistic case: contractual enforcement upheld, renegotiation is voluntary

- Pareto-improving renegotiation is a concern because what is Pareto optimal in period 2 may not be so in period 1.

- Question: why wait with renegotiation until start of period 2? why not right after buyer chooses among separating offers in period 1?

- *Sequential Pareto optimality* – the price in period 2 has to be sequentially optimal: $P_2 = v_H$ if $\beta(P_1) > \beta'$, $P_2 = v_L$ otherwise.

- This is identical to selling without commitment. Long-term contracting with renegotiation has the same outcome as the no-commitment case.

- *The renegotiation-proofness principle*: the solution is implemented through a renegotiation-proof contract. For example, when full separation is optimal: the buyer is offered a long-term contract with two options, either consuming in both periods at a price $v_H + \delta v_L$ or in period 2 only at a period-1 price of δv_L.

- Long-term contracting gets rid of the ratchet effect. So now selling and renting are similar always and equivalent to selling without commitment.

Multi-period regulation with asymmetric information

- Laffont and Tirole (1993)

- A natural monopoly produces a good with costs $c = \theta - e$, where θ is private information, $\theta \in \{\theta_L, \theta_H\}$, $\Delta\theta = \theta_H - \theta_L > 0$, $e > 0$ is effort, cost of effort $\psi(e) = e^2/2$.

- Government wants the good produced for the lowest possible payment $P = s + c$, where s is subsidy in excess of cost c. The firm's payoff is $s - \psi(e)$. First best: $e^* = 1$, $s = \psi(1) = 0.5$.

- Government can observe a firm's cost c but not its components θ and e. Government's prior belief that firm has a low θ is:

$$\beta_1 = \Pr(\theta = \theta_L).$$

- Single-period case
    - Contract: (s, c) – subsidy s received when observed cost is c. Contract menu: $\{(s_L, c_L), (s_H, c_H)\}$.
    - When a firm of type θ chooses contract (s, c), it picks effort $e = \theta - c$.
    - In order for type L to mimic type H, it must choose a lower effort than type H, $e_H - \Delta\theta$, in order to compensate for its lower θ to obtain the same cost c_H.
    - Participation constraints. Incentive constraints.
    - Solution involves $e_L = 1$ and $e_H = 1 - \frac{\beta_1}{1-\beta_1}\Delta\theta$.
    - Efficiency at the top, and underprovision of effort for the inefficient type, in order to reduce the rent of the efficient type.

- Two periods. Period 1 of length 1, period 2 of length δ.
    - A trick in order to allow $\delta > 1$.

- Full commitment. Long-term contract.
    - Contract menu $\{(s_{L1}, c_{L1}, s_{L2}, c_{L2}), (s_{H1}, c_{H1}, s_{H2}, c_{H2})\}$.
    - Solution: two-period replication of single-period contract menu.

- Renegotiation-proof long-term contract.
    - <u>Separating contract</u>: if $(s_{L1}, c_{L1}) \neq (s_{H1}, c_{H1})$, then period-2 outcomes must be efficient for both types – otherwise, Pereto-improving renegotiation would take place between firm and government.

        - Government's objective:

$$\min\{\beta_1[s_{L1} - e_{L1} + \delta(s_{L2} - e_{L2})] + (1 - \beta_1)[s_{H1} - e_{H1} + \delta(s_{H2} - e_{H2})]$$

        - binding constraints
            - inefficient type's participation constraint

$$s_{H1} - e_{H1}^2/2 + \delta(s_{H2} - e_{H2}^2/2) = 0$$

            - efficient type's incentive constraint

$$s_{L1} - e_{L1}^2/2 + \delta[s_{L2} - e_{L2}^2/2] = s_{H1} - (e_{H1} - \Delta\theta)^2/2 + \delta[s_{H2} - (e_{H2} - \Delta\theta)^2/2]$$

$$\Leftrightarrow$$

$$s_{L1} - e_{L1}^2/2 + \delta[s_{L2} - e_{L2}^2/2] = \Delta\theta[e_{H1} + \delta e_{H2} - (1 + \delta)\Delta\theta/2]$$

            - renegotiation-proofness constraints

$$e_{L2} = e_{H2} = 1$$

- Solution: same outcome in period 1 as with long-term contracts: $e_{L1} = 1$ and $e_{H1} = 1 - \frac{\beta_1}{1-\beta_1}\Delta\theta$.

- Because e_{H2} is higher than with full commitment, the efficient type now receives more rent.

- This rent concession is costly for government. But since it is proportional to δ, it is a little problem if δ is close to 0, i.e., if period 2 is very short.

o Pooling contracts: $(s_{L1}, c_{L1}) = (s_{H1}, c_{H1}) = (s_1, c_1)$

- Effort levels $e_H = \theta_H - c_1$ and $e_L = \theta_L - c_1$, implying that the efficient type now has the *lower* effort.

- Government beliefs unaltered after period 1, so renegotiation-proofness is not a constraint.

- Period-2 efforts as in the full-commitment case: $e_{L2} = 1$ and $e_{H2} = 1 - \frac{\beta_1}{1-\beta_1}\Delta\theta$.

- Period 1: underprovision of effort now for the efficient type, with $e_{H1} = 1$ and $e_{L1} = 1 - \Delta\theta$.

- Loss to government relative to the full-commitment case occurs in period 1, and so this loss is small if $1/\delta$ is small, that is, δ is large or period 2 is much longer than period 1.

- An analysis of *semiseparating* contracts reveals that full pooling can never be optimal, only asymptotically so as $\delta \to \infty$.

- Short-term contracts

  - Government's belief in period 2 that the firm has a low θ is β_2 – its value depends on first-period outcome.

  - Outcome in period 2: $e_{L2} = 1$ and $e_{H2} = 1 - \frac{\beta_2}{1-\beta_2}\Delta\theta$.

  - Full separation in period 1

    - There will be no rent to either type in period 2.

    - Incentive constraints may now bind for both types

    - Incentive constraint for low θ in period 1
      - all benefit from revealing type must come in period 2, while the benefit from disguising as high would come over both periods.

$$s_{L1} - e_{L1}^2/2 \geq s_{H1} - (e_{H1} - \Delta\theta)^2/2 + \delta[e_{H2}^2/2 - (e_{H2} - \Delta\theta)^2/2]$$

    - Incentive constraint for high θ in period 1: the high-θ firm must be given incentives to not mimicking low type and simply take the money and run

$$s_{H1} - e_{H1}^2/2 \geq s_{L1} - (e_{L1} + \Delta\theta)^2/2$$

  - More pooling than with long-term contracts and renegotiation

  - Even full pooling may be optimal.

Changing types

- Informed party's type change over time.

- The conclusion that enduring relationships are not beneficial does not hold anymore.

- Example: individual privately observing income shocks (or other shocks) over time.
    - o Intertemporal allocation of consumption improved by financial contracting.

A model of banking

- Diamond and Dybvig (1983)
- A continuum of ex ante identical consumers, living for 2 periods.
- Three dates: $t = 0, 1, 2$.
- Each consumer has 1 dollar to invest at date 0.
- Two projects available:
    - o a short-term project, yielding return r at date 1, which can be rolled over to yield r^2 at date 2, where $r \geq 1$.
    - o a long-term project yielding nothing at date 1 and $R > r^2$ at date 2.
    - o the long-term project may be liquidated at date 1, yielding liquidation value L.

- At date 0, consumers don't know their preferences. Type 1 consumers ($\theta = \theta_1$) are impatient and prefer consuming at date 1. Type 2 consumers ($\theta = \theta_2$) are patient and prefer consuming at date 2.

$$U(c_1, c_2, \theta) = \begin{Bmatrix} u(c_1 + \eta c_2), \text{if } \theta = \theta_1 \\ u(\mu c_1 + c_2), \text{if } \theta = \theta_2 \end{Bmatrix}$$

where $\eta < 1$, $\mu < 1$, $u' > 0$, $u'' < 0$.
    - o Long-term project better, but uncertainty may lead consumers to invest in short-term project instead.

- Ex ante probability of being a type-1 consumer is $\Pr(\theta = \theta_1) = \gamma$.

- From law of large numbers: there is a fraction γ of type-1 consumers in the economy.
  - A bank takes advantage of this regularity to invest in the long-term project despite consumers' preference risk.

- First-best solution
  - c_{it} is consumption per consumer of type i at date t.
  - Each consumer consumes at one date only: $c_{12}^* = c_{21}^* = 0$.
  - If $L \geq r$, then all is invested in long-term project, and an amount $y = c_{11}^*/L$ is liquidated at date 1.
  - If $L < r$, then $x = c_{11}^*\gamma/r$ is invested in the short-term project, the remainder in the long-term project.
  - Assume $L < r$ from now on.

- Second-best problem
  - Incentive constraints: a consumer must not prefer to mimic the other type.
    - Impatient consumer is not better off pretending to be patient by delaying consumption
      $$u(c_{11}^*) \geq u(\eta c_{22}^*)$$
    - Patient consumer is not better off pretending to be impatient:
      $$u(c_{22}^*) \geq u(rc_{11}^*)$$
      - The patient consumer does not need to consumer early in order to mimic impatient – it is sufficient to withdraw funds from the bank and reinvest them at a rate r.
  - The two incentive constraints are satisfied by the first-best solution only if
    $$\eta r < 1$$
  - This condition is necessary, but not sufficient.
  - The second-best solution may provide less than full insurance, with $c_{21} > 0$ (impatient consumers consuming early).

Bank vs market

- Financial intermediation may emerge endogenously in this economy.
    - Bank-based financial system: Consumers handing over their funds to a bank that places them in a suitable portfolio of the projects and offers consumers demand-deposit contracts that allow them, at date 1 when they know their preferences, to choose between the two consumption patterns $(c_{11}^{SB}, c_{12}^{SB})$ and $(c_{21}^{SB}, c_{22}^{SB})$.
    - Market-based financial system: Consumers investing in equity at date 0 that pays dividends equal to

      $$\gamma c_{11}^{SB} + (1-\gamma) c_{21}^{SB}$$

      at date 1, when consumers can trade dividends for (ex-dividend) shares in the stock market.
        - Equivalently: consumers handing over funds to mutual funds holding shares in publicly traded firms.

- The second-best solution would benefit from consumers not being able to reinvest withdrawn funds in period 1.
    - Optimal to provide insurance to consumers with nontraded instruments.
    - An argument in favour of a bank-based financial system.

- The bank's demand-deposit contract may give rise to bank runs.
    - If all patient consumers decide to mimic impatient consumers at date 1, the bank does not have enough money to meet withdrawal demands.